

III SEMESTER EXAMINATION, 2023 – 24
IIInd yr B.Tech. – (Civil Eng/CS&E/AI&ML/IT/EE/E&EE/ME/Robotics & Automation)
ADVANCE APPLIED MATHEMATICS

Max Marks: 100

Duration: 3:00 hrs

Note: - Attempt all questions. All Questions carry equal marks. In case of any ambiguity or missing data, the same may be assumed and state the assumption made in the answer.

Q 1.	<p>Answer any four parts of the following.</p> <p>a) Find the Laplace transform of the function $F(t) = (\sin t - \cos t)^2$.</p> <p>b) Find the Fourier sine transform of $\frac{1}{x}$.</p> <p>c) Find the third divided with arguments 2,4,9,10 of the function $f(x) = x^3 - 2x$.</p> <p>d) The first three moments about the origin are given by</p> $\mu_1 = \frac{n+1}{2}, \mu_2 = \frac{(n+1)(2n+1)}{6}, \mu_3 = \frac{n(n+1)^2}{4}$ <p>Examine the skewness of the data.</p> <p>e) For a group of 20 items, $\sum x_i = 200$, $\sum x_i^2 = 500$ and median 1.5, find Karl person's coefficient of skewness.</p> <p>f) Find $\frac{dy}{dx}$ at $x = 0.1$ from the following data</p> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">0.1</td> <td style="padding: 5px;">0.2</td> <td style="padding: 5px;">0.3</td> <td style="padding: 5px;">0.4</td> </tr> <tr> <td style="padding: 5px;">y</td> <td style="padding: 5px;">0.9975</td> <td style="padding: 5px;">0.9900</td> <td style="padding: 5px;">0.9776</td> <td style="padding: 5px;">0.9604</td> </tr> </table>	x	0.1	0.2	0.3	0.4	y	0.9975	0.9900	0.9776	0.9604	5x4=20
x	0.1	0.2	0.3	0.4								
y	0.9975	0.9900	0.9776	0.9604								
Q 2.	<p>Answer any four parts of the following.</p> <p>a) Find the inverse Laplace transform of $\frac{1}{9s^2 + 6s + 1}$.</p> <p>b) Find the Fourier cosine transform of $f(x) = e^{-ax}$</p> <p>c) Find the first four moments for the following individual data:</p> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">3</td> <td style="padding: 5px;">9</td> <td style="padding: 5px;">12</td> <td style="padding: 5px;">20</td> </tr> </table> <p>d) Find a root of the equation $x^3 - 4x - 9 = 0$. Using Bisection method is four stages.</p> <p>e) The first three moments of a distribution, about the value 2 of the variable are 1, 16 and -40. Show that the mean is 3, Variance is 15 and $\mu_3 = -86$.</p>	x	1	3	9	12	20	5x4=20				
x	1	3	9	12	20							

f) Use Trapezoidal rule to evaluate $\int_0^1 x^3 dx$ considering five intervals.

Q 3. Answer any two parts of the following.

10x2=20

a) State convolution theorem and hence find $L^{-1}\left[\frac{1}{s^3(s^2+1)}\right]$.

b) Using Laplace transform, solve the following equation

$$\frac{d^2x}{dt^2} + 9x = \cos 2t, \text{ if } x(0) = 1 \text{ and } x\left(\frac{\pi}{2}\right) = -1.$$

c) Use Fourier Sine transform to solve the equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$, under the condition

(i) $u(0, t) = 0$ (ii) $u(x, 0) = e^{-x}$ and (iii) $u(x, t)$ is bounded.

Q 4. Answer any two parts of the following.

10x2=20

a) Solve $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ for $x \geq 0, t \geq 0$ under the given condition $u = u_0$ at $x = 0, t > 0$ with initial condition $u(x, 0) = 0, x \geq 0$.

b) Find the root of the equation $\tan x + \tanh x = 0$, which lies in the interval (1.6, 3.0) correct to four significant digits using method of False position.

c) The function $y = f(x)$ is given at the point (7,3), (8,1), (9,1) and (10,9). Find the value of y for $x = 9.5$, using Lagrange's interpolation formula.

Q 5. Answer any two parts of the following.

10x2=20

a) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using (i) Simpson's one-third rule (ii) Simpson's three-eighth rule (iii) Trapezoidal rule

b) Calculate the coefficient of the Skewness from the following data:

Wage in rupees	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of labours	185	77	34	180	136	23	50

c) Use least square method to fit a curve of the form $y = ae^{bx}$ to the following data:

x	1	2	3	4	5	6
y	7.209	5.265	3.846	2.809	2.052	1.499