Sub Code: CST-002

ROLL NO

ODD SEMESTER EXAMINATION, 2023 – 24

2nd Year , B.Tech– (CS&E/AI&ML/IT)

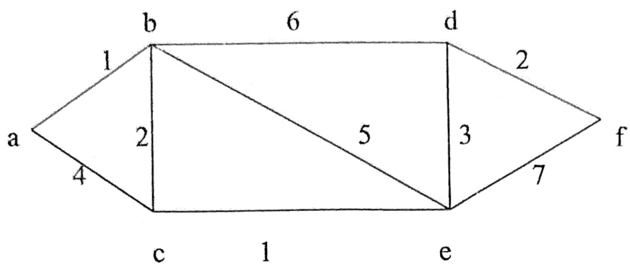
Discrete Structure

Max Marks: 100

Duration: 3:00 hrs

Note: - Attempt all questions. All Questions carry equal marks. In case of any ambiguity or missing data, the same may be assumed and state the assumption made in the answer.

Q 1.	Answer any four parts of the following. a) If A and B be finite sets and $f: A \to B$. Then show that (i) If f is injective, then $\|A\| \leq \|B\|$ (ii) If f is surjective, then $\|B\| \leq \|A\|$ b) Using mathematical induction method verify that $1^2 + 2^2 + 3^2 + \cdots \ldots \ldots n^2 = \frac{n(n+1)(2n+1)}{6}$ c) Let $f: A \to B$ and $g: B \to C$ be functions. If f and g are surjections, then show that $gof: A \to C$ is a surjection. d) Show that the mapping $f: R \to R$ be defined by $f(x) = ax + b$, where $a, b, x \in R, a \neq 0$ is invertible. e) For any three sets A, B and C, prove that $A - (B \cap C) = (A - B) \cup (A - C)$ f) Define the following: (i) Multisets (ii) Union of Multisets (iii) Sum of Multisets (iv) Intersection of Multisets (v) Difference of Multisets	5x4=20
Q 2.	Answer any four parts of the following. a) Prove that the proposition $\sim (p \wedge q) \vee q$ is a tautology. b) Prove that the intersection of two sublattices is a sublattice. c) Prove that two bounded lattices A and B are complemented if and only if $A \times B$ is complemented. d) Determine whether the following is a tautology, contingency and a contradiction: (i) $p \to (p \to q)$ (ii) $p \to (q \to p)$ (iii) $p \wedge \sim p$ e) Consider the poset $A = (\{1, 2, 3, 4, 6, 9, 12, 18, 36\}, /)$, find the greatest lower bound and the least upper bound of the sets $\{6, 18\}$ and $\{4, 6, 9\}$. f) If L be any lattice, then for any $a, b, c \in L$, prove that $a \vee (b \vee c) = (a \vee b) \vee c$.	5x4=20
Q 3.	Answer any two parts of the following. a) Solve the recurrence relation $a_{r+2} - 5\, a_{r+1} + 6\, a_r = 2$ by the method of generating functions with initial conditions $a_0 = 1$ and $a_1 = 2$. b) (i) How many 11 letter words can be formed using the letters of the word INSTITUTION. (ii) Prove that $C(n, r) = C(n-1, r) + C(n-1, r-1)$. c) Among the first 1000 positive integers, using principle of inclusion and exclusion, determine the integers which are not divisible by 5, nor by7, nor by 9.	10x2= 20
Q 4.	Answer any two parts of the following. a) If R_+ be the multiplicative group of all positive real numbers and R be the additive group of all real numbers. Show that the mapping $g: R_+ \to R$ defined by $g(x) = logx\ \forall\ x \in R_+$ is an isomorphism.	10x2= 20

	b) Prove that the set of all positive rational numbers under the binary operation $*$ defined by $a * b = \frac{ab}{2}$ is a group. c) Show that a commutative ring R is an integral domain if and only if for all $a,\ b,\ c \in R\ (a \neq 0)\ ab = ac \Rightarrow b = c$.	
Q 5.	Answer any two parts of the following. a) Apply Dijkstra's algorithm to the graph given below and find the shortest path from a to f :  b) If a linear graph has exactly one path between any two vertices. The linear graph is a tree and Conversely. Explain it. c) If G be a connected planar simple graph with e edges and v vertices. If r be the number of regions in a planar representations of G, then show that $r = e - v + 2$	10x2= 20

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